U.S. Productivity in Agriculture and R&D

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May 2005

1 Introduction

In this paper we analyze the impact of research and development (R& D) on productivity in the U.S. agricultural sector. Based on an extensive time series data, we estimate productivity using a Bennet-Bowley productivity index. This productivity indicator may be derived from the Luenberger productivity indicator which is in turn defined in terms of directional technology distance functions. These distance functions simultaneously credit expansions of outputs and contractions of inputs. Thus they are appropriate measures of total factor productivity in a profit maximization setting.

The Bennet-Bowley productivity indicator is an extremely useful research tool for dealing with the particular requirements of our task: namely, to construct a TFP measure for a multiple-multiple input technology. The data set includes two outputs, livestock and crops as well as four inputs including machinery, labor, fertilizer and land. The other part of our inquiry is to investigate the effect of research and development on the pattern of productivity growth. Here we employ time series techniques to relate our productivity series and time series data on R& D in U.S. agriculture. We find that: 1) we cannot reject the presence of a cointegrating relationship between the two series, 2) we cannot reject the

*We would like to thank Ken Carlaw and the participants in the 2005 NZESG meetings at University of Canterbury for their comments.

hypothesis that R&D does not Granger cause productivity change, and these series are related, and 3) productivity responds positively between four and ten periods after an R&D shock.

2 Constructing the Productivity Series

In this section we introduce a price dependent and a price independent productivity indicator, namely the Bennet-Bowley and Luenberger productivity indicators, respectively. We begin with the Luenberger productivity indicator[1] and the second is based on the Bennet-Bowley indicator (1920). By invoking the quadratic approximation lemma[2] and assuming a quadratic form for the distance function, one can prove that the Luenberger indicator can be expressed as a Bennet-Bowley indicator, which is the basis of our empirical work.[3]

As for notation, let $x \in \Re_+^N$ denote inputs and $y \in \Re_+^M$ denote outputs. The technology consists of all feasible input/output vectors,

$$T \; = \; \{(x, y) : x \text{ can produce } y\}. \tag{1}$$

Let $g \; = \; (g_x, g_y) \in \Re_+^{N+M}$ be a directional vector—a vector which determines the direction in which data are projected and technical efficiency is evaluated, then the technology directional distance function is defined as[4]

$$\vec{D}_T(x, y; g_x, g_y) \; = \; \sup\{\beta : (x - \beta g_x, y + \beta g_y) \in T\}. \tag{2}$$

Figure 1 illustrates.

The technology is labeled T, the directional vector is in the fourth quadrant indicating that inputs are to be contracted and outputs expanded. The distance function projects the input/output vector (x, y) onto the technology frontier at

[1] This indicator was introduced by Chambers (1996); some of its properties are discussed in Chambers, Färe and Grosskopf (1996).

[2] This lemma is due to Diewert (1976), see also Lau (1979).

[3] This was done by Chambers (1996,2002) and Balk (1998).

[4] This function was introduced by Luenberger (1992) under the name shortage function. See also Chambers, Chung and Färe (1998).

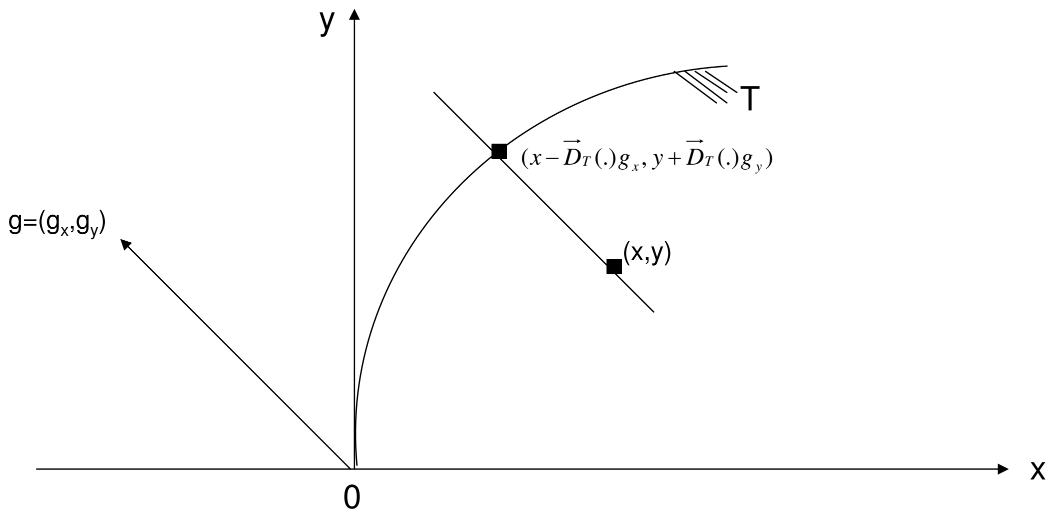

Figure 1: The Directional Technology Distance Function

$(x - \vec{D}_T(x, y; g_x, g_y)g_x, y + \vec{D}_T(x, y; g_x, g_y)g_y)$ in the direction $g = (g_x, g_y)$.

The distance function derives its properties from the technology and from its definition.[5] Here we focus on two distinguishing properties, namely the translation property

$$\vec{D}_T(x - \alpha g_x, y + \alpha g_y; g) = \vec{D}_T(x, y; g_x, g_y) - \alpha, \alpha \in \Re \tag{3}$$

and the representation property

$$\vec{D}_T(x, y; g_x, g_y) \geqq 0 \text{ if and only if } (x, y) \in T. \tag{4}$$

The translation property is the additive analog of the homogeneity property of the Shephard distance functions. It is a consequence of the definition of the directional distance function.

The representation property tells us that the distance function is a complete representation of the technology T. Noting this and that

[5]See Luenberger (1992) or Chambers, Chung and Färe (1998).

$$(x - \vec{D}_T(x, y; g_x, g_y)g_x, y + \vec{D}_T(x, y; g_x, g_y)g_y) \in T \tag{5}$$

one may define the profit function in terms of the distance function and retrieve the distance function from the profit function. Put differently, the profit function and the directional technology distance functions are dual to each other. Let $p \in \Re_+^M$ be output prices and $w \in \Re_+^N$ be input prices, then

$$\Pi(p, w) \;=\; \max_{x,y}\{p(y + \vec{D}_T(x, y; g_x, g_y)g_y) - w((x - \vec{D}_T(x, y; g_x, g_y)g_x)\} \tag{6}$$

and

$$\vec{D}_T(x, y; g_x, g_y) \;=\; \min_{p,w}\{\frac{\Pi(p, w) - (py - wx)}{pg_y + wg_x})\}. \tag{7}$$

The first part shows that the profit function is defined in terms of the distance function and the second part shows that the distance function can be recovered from the profit function.

If we assume differentiability then

$$(\bigtriangledown_x \vec{D}_T(x, y; g_x, g_y), \bigtriangledown_y \vec{D}_T(x, y; g_x, g_y)) \;=\; argmin_{w,p}\{\frac{\Pi(p, w) - (py - wx)}{pg_y + wg_x}\} \tag{8}$$

are the (deflated) shadow prices of inputs and outputs, respectively.

Now define the Shephard (1970) output distance function as

$$D_o(x, y) \;=\; \sup\{\theta : (x, y/\theta) \in T\}, \tag{9}$$

then its relation to the directional distance function is

$$\vec{D}_T(x, y; 0, y) \;=\; \frac{1}{D_o(x, y)} - 1, \tag{10}$$

i.e., by setting the directional vector $g = (0, y)$, we see that the Shephard output distance function is a special case of the directional technology distance function.

Next we introduce the Luenberger productivity indicator and assume that (x^t, y^t) and (x^{t+1}, y^{t+1}) are two observations of data. If we evaluate these data points relative to the period t technology we may define the t-period Luenberger indicator as

$$\pounds^t \;=\; \vec{D}_{T^t}(x^t, y^t; g) - \vec{D}_{T^t}(x^{t+1}, y^{t+1}; g) \tag{11}$$

where the distance functions are given by

$$\vec{D}_{T^t}(x^\tau, y^\tau; g) \;=\; \sup\{\beta : (x^\tau - \beta g_x, y^\tau + \beta g_y) \in T^t\}, \tau = t, t+1. \tag{12}$$

Figure 2 illustrates.

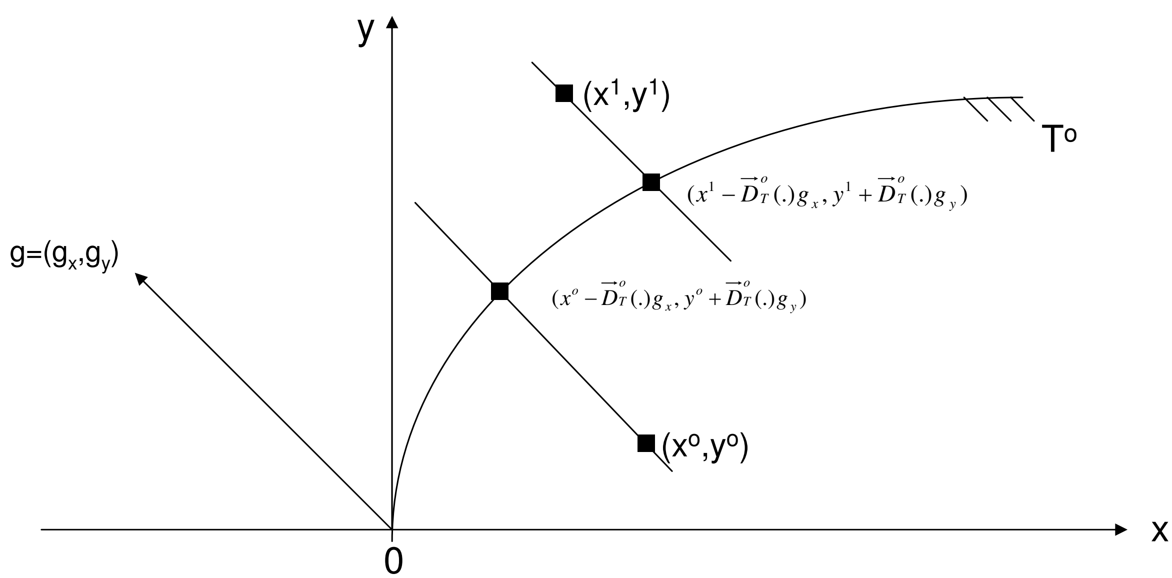

Figure 2: The Base-Period Luenberger Productivity Indicator

The t-period technology T^t is the area between the curved line and the x-axis (we set t=0 in Figure 2). The directional vector is found in the fourth quadrant, where inputs are to be contracted and outputs expanded. The distance function projects the two observations (x^t, y^t) and (x^{t+1}, y^{t+1}) onto the frontier of T^t.

The data (x^τ, y^τ) may also be projected onto the frontier of the $t+1$ technology T^{t+1}, creating a $t+1$ period Luenberger productivity indicator

$$\pounds^{t+1} \;=\; \vec{D}_{T^{t+1}}(x^t, y^t; g) - \vec{D}_{T^{t+1}}(x^{t+1}, y^{t+1}; g). \tag{13}$$

Since the choice of t or $t+1$ for the reference technology is arbitrary, we instead use the arithmetic mean of the two for our indicator here,[6]

$$\pounds_t^{t+1} \;=\; \frac{1}{2}(\pounds^t + \pounds^{t+1}). \tag{14}$$

Before we introduce the time dependent Bennet-Bowley productivity indicator, we show how the Luenberger indicator is related to the more familiar Malmquist productivity index. The t-period Malmquist productivity index is defined in terms of the Shephard output distance function as

$$M_o^t \;=\; D_o^t(x^{t+1}, y^{t+1})/D_o^t(x^t, y^t). \tag{15}$$

This index is a ratio between Shephard distance functions, whereas the Luenberger indicator is a difference between directional distance functions. Since the distance functions are related, we can also derive a relationship between the Malmquist index and the Luenberger indicator. The difference in structure (ratio versus differences) is addressed by appealing to the logarithmic mean,
which states that for any positive real numbers a and b,[7]

$$L(a,b) \;=\; (a-b)/\ln(a/b), L(a,a) \;=\; a. \tag{16}$$

In order to relate Malmquist and Luenberger we also need to assume that
$\vec{D}_{T^\tau}(x^\tau, y^\tau; 0, y^\tau) = 0, \tau = t, t+1$ and that $y^{t+1} > 0$. These assumptions imply that

$$\vec{D}_{T^t}(x^t, y^t; 0, y^{t+1}) = 0 \tag{17}$$

see Balk, et al (2005). If we choose $g \;=\; (0, y^{t+1})$, account for the relation between the Shephard and directional distance functions and the assumptions above we have the following

[6] Färe, Grosskopf and Margaritis (2005) provide a condition for $\pounds^t$ to equal $\pounds^{t+1}$. It states that the distance function must be additively separable in inputs/outputs and time.

[7] See Balk (2003), who attributed this index to Törnqvist.

$$\pounds^t = \frac{1}{D_o^t(x^t, y^t)} - \frac{1}{D_o^t(x^{t+1}, y^{t+1})}. \tag{18}$$

Applying the logarithmic mean to this expression yields

$$\left(\ln \frac{1}{D_o^t(x^t, y^t)} - \ln \frac{1}{D_o^t(x^{t+1}, y^{t+1})}\right) L\left(\frac{1}{D_o^t(x^t, y^t)}, \frac{1}{D_o^t(x^{t+1}, y^{t+1})}\right) = \pounds^t \tag{19}$$

which yields

$$M^t = e^{\pounds^t / L(\frac{1}{D_o^t(x^t, y^t)}, \frac{1}{D_o^t(x^{t+1}, y^{t+1})})}. \tag{20}$$

The corresponding Luenberger indicator

$$\pounds_t^{t+1} = \frac{1}{2}(\pounds^t + \pounds^{t+1}) \tag{21}$$

can then be related to the geometric mean Malmquist index

$$M_t^{t+1} = (M^t \cdot M^{t+1})^{1/2} \tag{22}$$

as[8]

$$M_t^{t+1} = e^{1/2\left(\pounds^t / L(\frac{1}{D_o^t(x^t, y^t)}, \frac{1}{D_o^t(x^{t+1}, y^{t+1})}) + \pounds^{t+1} / L(\frac{1}{D_o^{t+1}(x^t, y^t)}, \frac{1}{D_o^{t+1}(x^{t+1}, y^{t+1})})\right)}. \tag{23}$$

Next we introduce the Bennet-Bowley productivity indicator. Recall that a function $F : \Re^J \rightarrow \Re$ is quadratic if

$$F(q) = \alpha_o + \sum_{j=1}^{J} \alpha_j q_j + \sum_{i=1}^{J} \sum_{j=1}^{J} \alpha_{ij} q_i q_j. \tag{24}$$

Assume that there are two q vectors, q^1 and q^o, then the quadratic lemma states that

$$F(q^1) - F(q^o) = (1/2)[\bigtriangledown F(q^o) + \bigtriangledown F(q^1)][q^1 - q^o]. \tag{25}$$

Assume that the directional technology distance function is quadratic,[9] and assume that the second order terms are time independent, then the Luenberger indicator $\pounds_t^{t+1}$ can be

[8]This was first shown by Balk, Färe, Grosskopf and Margaritis (2005).

[9]The quadratic form readily allows for the translation property of the directional distance function, see Chambers (2002).

modeled as a price dependent Bennet-Bowley productivity indicator[10] i.e,

$$
\begin{aligned}
(BB)_t^{t+1} &= \frac{1}{2}\left[\frac{p^t}{p^t g_y + w^t g_x} + \frac{p^{t+1}}{p^{t+1} g_y + w^{t+1} g_x}\right][y^{t+1} - y^t] \\
&\quad -\frac{1}{2}\left[\frac{w^t}{p^t g_y + w^t g_x} + \frac{w^{t+1}}{p^{t+1} g_y + w^{t+1} g_x}\right][x^{t+1} - x^t] \\
&= \frac{1}{2}\sum_{m=1}^{M}\left[\frac{p_m^t}{p^t g_y + w^t g_x} + \frac{p_m^{t+1}}{p^{t+1} g_y + w^{t+1} g_x}\right][y_m^{t+1} - y_m^t] \\
&\quad -\frac{1}{2}\sum_{n=1}^{N}\left[\frac{w_n^t}{p^t g_y + w^t g_x} + \frac{w_n^{t+1}}{p^{t+1} g_y + w^{t+1} g_x}\right][x_n^{t+1} - x_n^t] \\
&= \pounds_t^{t+1}.
\end{aligned}
\tag{26}
$$

Note that if $g_y = 1$ and $g_x = 1$, then the weights

$$
\frac{1}{2}\left(\frac{p^t}{\sum_{m=1}^{M} p_m^t + \sum_{n=1}^{N} w_n^t} + \frac{w^t}{\sum_{m=1}^{M} p_m^t + \sum_{n=1}^{N} w_n^t}\right)
\tag{27}
$$

sum to one for each t, as they should. We use this insight to set $g_y = 1, g_x = 1$ in our empirical application below.

Although we do not estimate the price independent productivity series—the Luenberger productivity indicator—here, one may do so using activity analysis or Data Envelopment Analysis (DEA). This method is especially useful in a panel data setting, since the cross-sections may be used to help to identify efficiency change and technical change components of total factor productivity. In the time series context that is difficult, thus our empirical application uses the Bennet-Bowley 'approximation' of the Luenberger indicator. In the time series context, one could use DEA to construct one reference technology from the entire data set. Thus the observations $k = 1, \ldots, K$ are the time periods $t = 1, \ldots, \bar{t}$, with $g_y = 1, g_x = -1$. For each t' one could compute

[10] See Chambers (1996,2002) or Balk (1998). The derivation requires formulation of $\vec{D}_T(x, y; g_x, g_y)$ as a quadratic function, the quadratic lemma and the fact that the derivatives are shadow prices.

$$
\begin{aligned}
\vec{D}_{T^{\bar{t}}}(x^t, y^t; 1) &= \max \beta \qquad (28) \\
s.t. \quad & \sum_{t=1}^{\bar{t}} z_t y_{tm} \geqq y_{t'm} + \beta, m = 1, \ldots, M \\
& \sum_{t=1}^{\bar{t}} z_t x_{tn} \leqq x_{t'n} - \beta, n = 1, \ldots, N \\
& z_t \geqq 0, t = 1, \ldots, \bar{t}.
\end{aligned}
$$

In this cawse, each linear programming problem here would be solved for a constant returns to scale, strongly disposable technology.

Based on these distance functions we could construct the Luenberger indicator for our single, time series technology which we refer to as $T^{\bar{t}}$, as

$$
\begin{aligned}
\pounds_t^{t+1} &= \frac{1}{2}\left(\vec{D}_{T^{\bar{t}}}(x^t, y^t; 1) - \vec{D}_{T^{\bar{t}}}(x^{t+1}, y^{t+1}; 1)\right) \qquad (29) \\
&+ \frac{1}{2}\left(\vec{D}_{T^{\bar{t}}}(x^t, y^t; 1) - \vec{D}_{T^{\bar{t}}}(x^{t+1}, y^{t+1}; 1)\right) \\
&= \vec{D}_{T^{\bar{t}}}(x^t, y^t; 1) - \vec{D}_{T^{\bar{t}}}(x^{t+1}, y^{t+1}; 1).
\end{aligned}
$$

Thus a primal productivity series could be estimated by substituting adjacent periods of the linear programming problem in (28) into (29). Here we focus instead on the price dependent series which we compute by directly substituting the relevant data into the Bennet-Bowley index in (26).

3 Results

We report the results for the Bennet-Bowley productivity indicator. We construct this indicator using data on two outputs (livestock and crops) and four inputs (machinery, labor, fertilizer, and land) over the 1910-1990 periods, see Thirtle, et al, (2002). The cumulative Bennet-Bowley indicator and private research and development expenditure indexes are displayed in Figure 3.

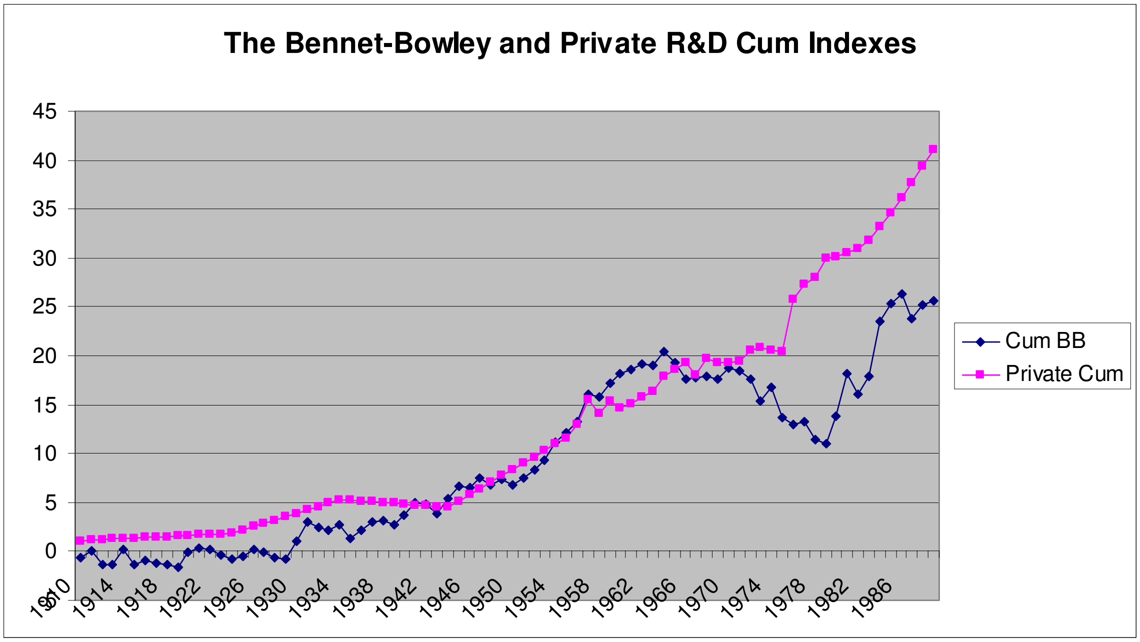

Figure 3: The Cumulative Bennet-Bowley and R&D Indexes

The average rate of productivity growth as measured by the Bennet-Bowley indicator is .32 percent per year over the sample period. The BB indicator and R&D index track each other closely over most of the sample period with the exception of the years between 1971 and 1979 when productivity growth declines, whereas research and development expenditures continue to grow.

We evaluate this relationship between the productivity indicator and R&D index more formally using standard tools of time series analysis. Both series appear to be non-stationary unit root processes. The ADF values are -2.039 and 0.461 for the Bennet-Bowley indicator and the R&D index, respectively. The associated p-values are .571 and .999. There is no evidence to suggest the presence of a second unit root in either series.

Next we investigate whether the two series are driven by a common stochastic trend. The results of the Johansen cointegration trace test give a trace statistic of 16.07 (p-value=.041); 3.15 (p-value=0.076) for testing the hypotheses of zero and at most one cointegrating vector respectively. Therefore, we cannot reject the presence of a cointegrating relationship between the two series.

Granger causality tests indicate one-way causality from R&D to productivity. Specifically we cannot reject the hypothesis that productivity does not Granger cause R&D (F=0.917, p=.489) whereas we reject the hypothesis that R&D does not Granger cause productivity change (F=3.387, p=.006).

Evidence on the dynamics of the relationship between the series is obtained from the estimation of a VEC model. The error-correction model equation for productivity is

$$\begin{aligned} \triangle BB = const. \quad & \underset{(-3.595)}{-.03ec(-1)} + \underset{(3.191)}{.570\triangle RD(-4)} \\ + \quad & \underset{(3.02)}{.547\triangle RD(-5)} + \underset{(4.109)}{.729\triangle RD(-8)} \\ + \quad & \underset{(1.396)}{.157\triangle BB(-4)} + \underset{(1.567)}{.179\triangle BB(-5)} \\ - \quad & \underset{(1.869)}{.245\triangle BB(-8)} \end{aligned} \tag{30}$$

with $R^2 = .48$

Figure 4 presents results of impulse response analysis over 16 periods. The lower left hand figure shows that productivity responds positively between four and ten periods after an R&D shock.

4 Summing Up

In this paper we investigate the relationship between productivity growth and R&D expenditure in U.S. agriculture over the 1910-1990 period. We estimate total factor productivity growth using a Bennet-Bowley productivity indicator. This may be thought of

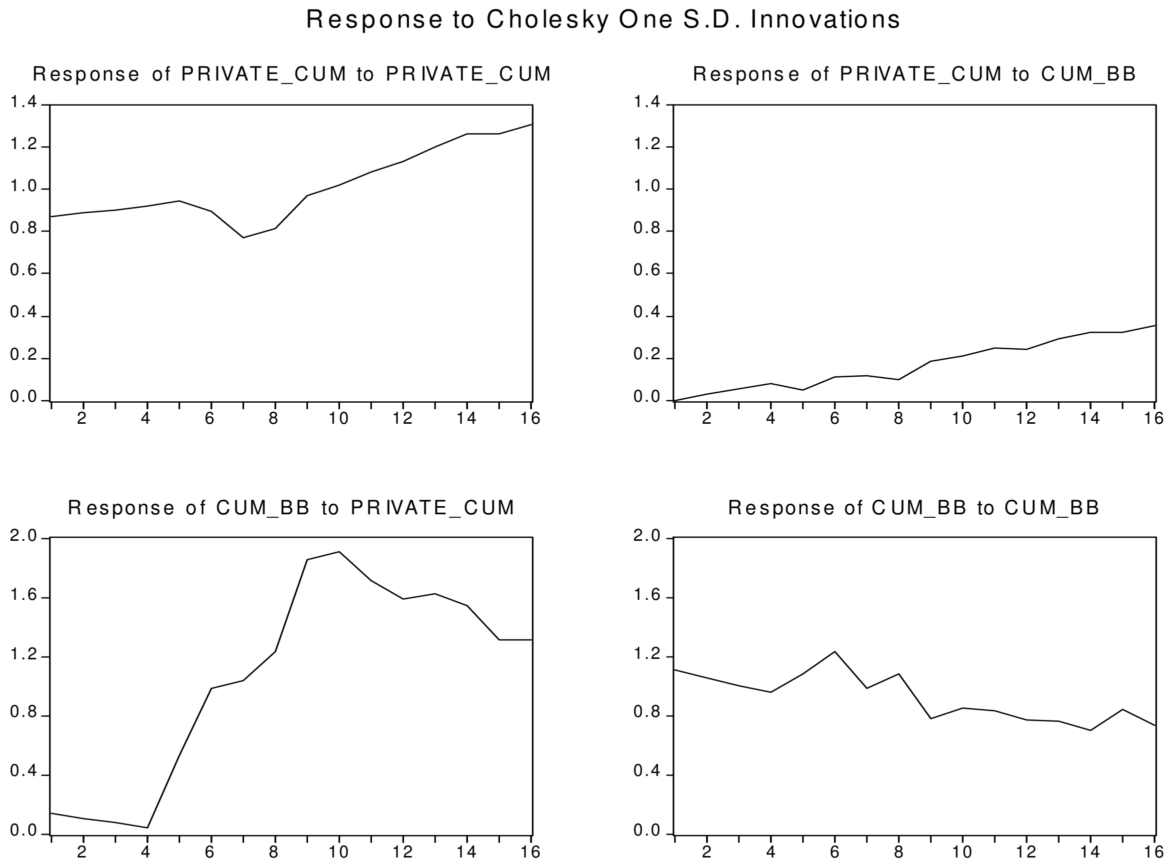

Figure 4: Impulse Response

as an approximation of the Luenberger productivity indicator, which is constructed from directional technology distance functions which represent technology without requiring price information. These are dual to the profit function, which we argue makes them especially useful in the agricultural setting.

The data we use is an extensive time series, with multiple outputs and inputs. Since time-series data does not allow us to exploit the ability of the Luenberger productivity indicator to identify its components of technical change and efficiency change, we compute its approximation, the Bennet-Bowley indicator.

Overall, our preliminary results suggest that U.S. agriculture realized positive productivity growth over the 1910-1990 period, and this growth is at least in part 'caused' by R&D expenditures over that period, with a lag of between four and ten periods after an R&D shock.

We would like to investigate these relationships with state level data. With such panel data, we could look directly at the relationship between R&D and innovation (technical change) and efficiency change (diffusion or catching up).

References

[1] Balk, B.M. (2003), The Residual: On Monitoring and Benchmarking Firms, Industries and Economies with Respect to Productivity, *Journal of Productivity Analysis* 20, 5-47.

[2] Balk B.M., R. Färe, S. Grosskopf and D. Margaritis (2005). The Equivalence Between the Luenberger Productivity Indicator and the Malmquist Productivity Index, mimeo.

[3] Bennet, T. L. (1920) 'The Theory of Measurement of Changes in Cost of Living. *Journal of the Royal Statistical Society* 83, 455-462.

[4] Chambers, R. G. (1996) A New Look at Exact Input, Output, Productivity and Technical Change Measurement, mimeo, Department of Agricultural and Resource Economics, University of Maryland, College Park.

[5] Chambers, R. G., Y. Chung, and R. Färe (1998). Profit, Directional Distance Functions and Nerlovian Efficiency. *Journal of Optimization Theory and Applications* 95:2, 351-364.

[6] Chambers, R., R. Färe and S. Grosskopf (1996), Productivity Growth in APEC Countries, *Pacific Economic Review*, 1, 181-190.

[7] Färe, R., S. Grosskopf and D. Margaritis (2005), Productivity and Efficiency: Malmquist and More, mimeo.

[8] Lau, L.J. (1979) On Exact Index Numbers. *The Review of Economics and Statistics* 61, 73-82.

[9] Luenberger, D.G. (1992) New Optimality Principles for Economic Efficiency and Equilibrium, *Journal of Optimization Theory and Applications* 75, 221-264.

[10] Shephard, R.W. (1970). *Theory of Cost and Production Functions.* Princeton, N.J.: Princeton University Press.

[11] Thirtle, C.G., D.E. Schimmelpfennig and R.E. Townsend (2002), Induced Innovation in U.S. Agriculture, 1880-1990: Time-Series Tests and an Error Correction Model, *American Journal of Agricultural Economics* 84:3, 598-614.

[12] Törnqvist, L., P. Vartia and Y.O. Vartia. (1985), How Should Relative Changes be Measured?, *The American Statistician* 39, 43-46.